Refresher for Vectors of Three Dimensions

[1] The <u>mathematical notation</u> used for a vector is a symbol with an arrow on top, like vector v below:

$$\vec{v}$$

[2] In the simulation program available on the web the vectors used are all <u>three dimensional</u>, e.g.

$$\vec{V} = \left(V_x, V_y, V_z \right)$$

[3] The <u>magnitude of a vector</u> is the length of a vector and calculated and represented as follows:

$$\left| \vec{V} \right| = \sqrt{V_x^2 + V_y^2 + V_z^2}$$

This should be quite familiar as it is just the Pythagorean Theorem applied to vectors. The magnitude is the straight line distance from the origin to the point in space given by the address $\left(V_x, V_y, V_z \right)$.

[4] A special kind of vector is called the <u>unit vector</u>. When you compute the magnitude of a unit vector the result is equal to one, hence the name – unit vector.

When you take any vector and divide each of its components by its magnitude you get a unit vector which points in the same direction as the original vector. Below is the proof for a vector of three dimensions.

Let's start with the same vector v from above.

$$\vec{V} = \left(V_x, V_y, V_z \right)$$

Again, the magnitude of vector v is:

$$\left| \vec{V} \right| = \sqrt{V_x^2 + V_y^2 + V_z^2}$$

So, to calculate the unit vector we divide each component by the magnitude above, or:

$$\left(\frac{V_x}{\sqrt{V_x^2 + V_y^2 + V_z^2}}, \frac{V_y}{\sqrt{V_x^2 + V_y^2 + V_z^2}}, \frac{V_z}{\sqrt{V_x^2 + V_y^2 + V_z^2}} \right)$$

To verify that this vector is of unit length we now calculate its magnitude, which is the same as the sum of the squares of each of the components, or

$$\left(\frac{V_x}{\sqrt{V_x{}^2 + V_y{}^2 + V_z{}^2}}\right)^2 + \left(\frac{V_y}{\sqrt{V_x{}^2 + V_y{}^2 + V_z{}^2}}\right)^2 + \left(\frac{V_z}{\sqrt{V_x{}^2 + V_y{}^2 + V_z{}^2}}\right)^2$$

When we apply the square to numerator and denominator

$$\frac{V_x{}^2}{\left(\sqrt{V_x{}^2 + V_y{}^2 + V_z{}^2}\right)^2} + \frac{V_y{}^2}{\left(\sqrt{V_x{}^2 + V_y{}^2 + V_z{}^2}\right)^2} + \frac{V_z{}^2}{\left(\sqrt{V_x{}^2 + V_y{}^2 + V_z{}^2}\right)^2}$$

Then, combining fractions over a common denominator shows that the numerator and the denominator are both equal – so the vector is of unit length.

$$\frac{V_x{}^2 + V_y{}^2 + V_z{}^2}{\left(\sqrt{V_x{}^2 + V_y{}^2 + V_z{}^2}\right)^2} = \frac{V_x{}^2 + V_y{}^2 + V_z{}^2}{V_x{}^2 + V_y{}^2 + V_z{}^2} = 1$$

[5] The next concept is that of <u>dot product</u>. There are two ways to define the dot product, the one we will be using is this one:

$$\vec{A} \cdot \vec{B} = \left(A_x, A_y, A_z\right) \cdot \left(B_x, B_y, B_z\right) = A_x B_x + A_y B_y + A_z B_z$$

[6] The dot product is used in determining the <u>projection</u> of one vector on another. In the figure below we see the yellow vector projecting on to the white vector. The length of the yellow vector on the white vector is taken with a line that starts at the end of the yellow vector and reaches the white vector at a right angle.
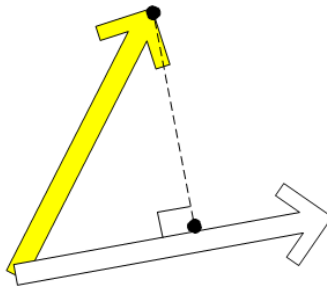
Figure 1 - Projecting the Yellow Vector on to the White Vector

Calculating the projection of one vector $\vec{A}$ on to another vector $\vec{B}$ involves performing the dot product of vector $\vec{A}$ to the unit vector of the second vector $\vec{B}$. The mathematical formulation is:

$$\vec{A} \cdot \left(\frac{\vec{B}}{|\vec{B}|} \right)$$

Projecting vector $\vec{A}$ onto $\vec{B}$ is equivalent to calculating the component of vector $\vec{A}$ parallel to vector $\vec{B}$.

[7] Any vector can be broken into two components, one that is perpendicular to another vector, and one component that is parallel to it. This is the mathematical formulation of breaking vector $\vec{F}$ into its perpendicular and parallel components.

$$\vec{F} = \vec{F_\perp} + \vec{F_=}$$

This is used in explaining the relativistic adjustments to gravitational force in the simulation.